WORCESTER POLYTECHNIC INSTITUTE

32nd INVITATIONAL MATH MEET

October 15, 2019

INDIVIDUAL EXAM with Answers

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes long. Questions 1-4 are each worth 1 point. Questions 5-8 are each worth 2 points. Questions 9-11 are each worth 3 points. Calculators and other electronics **MAY NOT** be used.

1 Point Each:

1. What are all the nonnegative integers x for which $\mathbf{(x-6)(x+14)}$ is a perfect square?
   ans: 6 , 22

2. A cow on 30' rope is hooked to a corner of a 10' x 20' barn.
   How many square feet of grazing area does she have? **ans:** 800π ft^2

3. Assume that all of the following are nxn invertible matrices: **A,B,C,F,X** . Solve the following equation for X: $\mathbf{A^2B^{-1}XC + F = I_n}$. ($\mathbf{I}_n$ denotes the nxn identity matrix)
   ans: $BA^{-2}(I_n - F)C^{-1}$

4. Consider the circle $x^2 + y^2 = 16$ and the ellipse $x^2/16 + y^2/9=1$. What is the area inside the circle and outside the ellipse?
   ans: 4π

2 Points Each:

5. If x is a real number such that $2x^3 + 4x^2 + 6x + 8 = 2468$, what is the value of $x^3 + 9x^2 + 8x + 8$??
   ans: 1988

6. What is the area of the smallest circle centered at the origin which touches both branches of the hyperbola **xy = 2** ?
   ans: 4π

7. Please evaluate
$$\sum_{j=2}^{5}\sum_{i=1}^{6}(ji^2+2)$$
   ans: 1322

8. In a certain school, 60% of the students have a dog at home. Suppose that 8 students are sampled . What is the probability that exactly 5 have a dog at home? (no need to simplify your resulting answer)
   ans: $(8!/(5!3!))^* (0.6)^5 (0.4)^3$

3 Points Each:

9. Consider the infinite series $1+\frac{i}{2}-\frac{1}{4}-\frac{i}{8}+\frac{1}{16}+\frac{i}{32}\ldots$ where $i=\sqrt{-1}$ which converges to a point in the complex plane. If the distance of that point from the origin is $\frac{a\sqrt{5}}{b}$ find a and b . **ans:** a=2 b = 5

10. What is the value of k for which

$$\frac{2000!}{1000!}=k(1x3x5x7x\ldots x\ 1997\ x\ 1999) \quad ?$$

ans: $k = 2^{1000}$

11. How many **0**'s are at the end of **62!** **ans**: 14